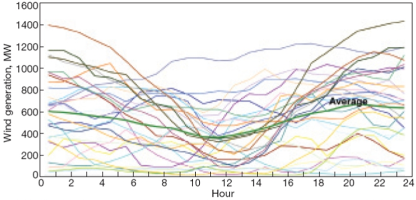
Representing storage and demand response

Josh Taylor

University of Toronto

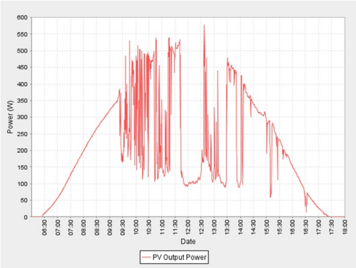
The need for flexibility in power systems

Renewable energy is intermittent and unpredictable.



7. Wind generation can vary dramatically from day to day. Each curve above illustrates the wind generation profile for one day in April 2009. Knowing what to expect from wind when lining up resources a day ahead is challenging for grid operators like the CalISO, which developed this chart based on its system data

24 hour wind power production



24 hour solar power production

Intermittency demands new sources of flexibility:

- Energy storage
- Loads (demand response)

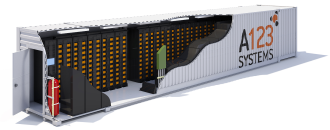
Energy Storage

Capabilities

- Load-shifting/arbitrage, frequency & voltage regulation, curtailment
- Natural inventory control formulation.[a]

[a] J.A. Taylor, D.S. Callaway, and K. Poolla. "Competitive energy storage in the presence of renewables". In: *Power Systems, IEEE Transactions on* 28.2 (May 2013), pp. 985–996. DOI: 10.1109/TPWRS.2012.2210573.

- Pumped hydro
- Batteries
- Flywheels, compressed air, supercapacitors...



A123 battery storage

Demand response, “supply following”

Example

5 MW drop in solar output is balanced by 3,000 residential AC’s turning off (for financial compensation).

Uses:

- Curtailment
- Load shifting
- Frequency regulation

Sources:

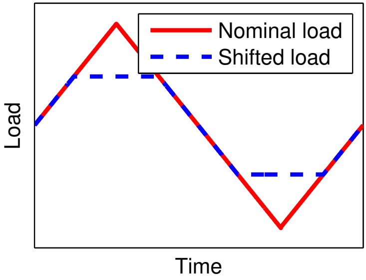
- Residential appliances, building HVAC, industrial
- Electric vehicles

Samsung smart air conditioner

Functionalities of storage and demand response

Load shifting / Peak shaving

- Inject during high demand, extract power during low demand
- Buy low, sell high
- *Time scale*: hours



This talk

How can we better fit storage and demand response in power systems?

- **Part 1:** Representing storage in markets.
- **Part 2:** Representing demand response similar to (but not the same as) storage.

Basics

- Generation (assets) sells to system operator.
- Loads buy from system operator.
- Prices are dual variables of economic dispatch / optimal power flow.
- *Time scale:* Hourly, 5 min

Motivation: conceptual

The analogy between transmission and storage

- Transmission moves power spatially, storage forward in time.
- Large upfront cost, inexpensive operation
- Hard power capacity limits (storage also has energy)

Transmission economics

- Lines do not buy power at one end and sell at the other (spatial arbitrage).
- Commonly financed with Financial Transmission Rights.

Should storage buy and sell power at nodal prices?

- If so, storage profits through intertemporal arbitrage. Case closed.
- If not, we need Financial Storage Rights. Call this Passive Storage.

Approach

1. Financial Transmission Rights are parametrized by dual multipliers from optimal power flow.
2. Storage is modeled in multiperiod optimal power flow (MOPF).
3. Dual multipliers from MOPF $\longrightarrow$ Financial Storage Rights.

Start with (1) for intuition.

Optimal power flow and nodal prices

$$\text{Minimize generation cost:} \quad \sum_i F_i(P_i) \quad \textit{subject to}$$

$$\text{Nodal power balance:} \quad P_i = \sum_j P_{ij} \tag{1}$$

$$\text{Transmission capacity:} \quad P_{ij} = B_{ij}(\theta_i - \theta_j) \leq L_{ij} \tag{2}$$

Lagrange duality:

- λ_i is the multiplier of (1).
- μ_{ij} is the multiplier of (2).

Nodal AKA locational marginal pricing

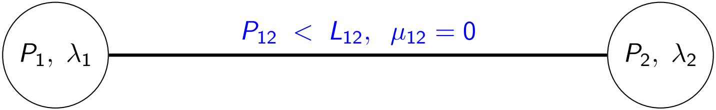
The load/generator at node *i* buys/sells P_i at price λ_i from the *System Operator (SO)*.

- Rigorous foundation in microeconomics
- "Successful" history in communication, transportation, capitalism

Transmission congestion in a two-node network

Node one sells P_1 to SO at λ_1, node two buys P_2 at λ_2 from SO.

Uncongested case:

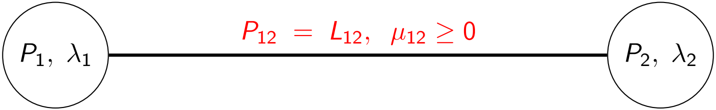


- $\lambda_2 - \lambda_1 = \mu_{12} = 0$
- SO budget: $\lambda_2 P_2 - \lambda_1 P_1 = 0$

Transmission congestion in a two-node network

Node one sells P_1 to SO at λ_1, node two buys P_2 at λ_2 from SO.

Congested case:



- $\lambda_2 - \lambda_1 = \mu_{12} \geq 0$
- SO budget: $\lambda_2 P_2 - \lambda_1 P_1 \geq 0$

Transmission congestion

Consequences of transmission congestion:

- System operator makes money (undesirable).
- In practice, $\lambda_2 >> \lambda_1$ (price spiking breaks load's bank account).
- Generators shortchanged.

Arithmetic with KKT yields:[1]

$$\underbrace{\underset{i}{}\lambda_i P_i}_{\text{SO budget}} + \underbrace{\underset{ij}{}\mu_{ij} L_{ij}}_{???} = 0$$

What does the latter term tell us?

[1] Felix Wu et al. "Folk theorems on transmission access: Proofs and counterexamples". In: *Journal of Regulatory Economics* 10 (1 1996), pp. 5–23.

Transmission rights

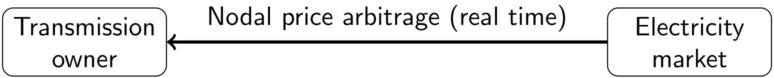
The SO has a budget surplus of $\sum_{ij} \mu_{ij} L_{ij}$.

Definition: The holder of a Flowgate Transmission Right is entitled to collect $\mu_{ij} L_{ij}^k$, $0 \leq L_{ij}^k \leq L_{ij}$ from SO.[2]

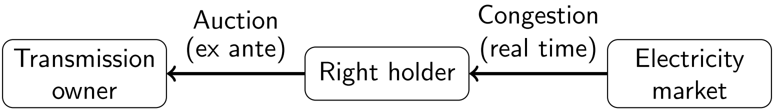
- If $\sum_k L_{ij}^k = L_{ij}$ for all ij, flowgates balance SO budget.
- Generators capture more profit, loads *hedge* against price spikes.

[2] Hung-Po Chao and Stephen Peck. "A market mechanism for electric power transmission". English. In: *Journal of Regulatory Economics* 10.1 (1996), pp. 25–59. DOI: 10.1007/BF00133357.

Revenue paths

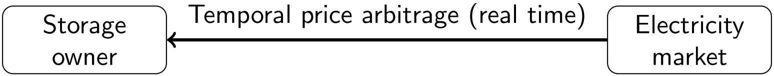


Hypothetical revenue path: Spatial arbitrage

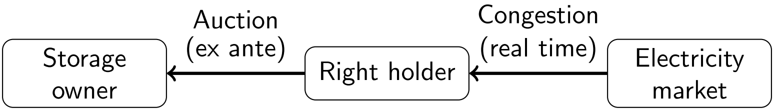


Actual revenue path: Transmission Rights

Which makes more sense for storage?



Option 1: Temporal arbitrage



Option 2: Storage rights *(passive storage)*

Some justification for rights

- Storage is like transmission, same justifications apply.
- Already under consideration by PJM.[a]

[a] M. Abdurrahman et al. *Energy Storage as a Transmission Asset*. Tech. rep. PJM Interconnection and Xtreme Power, 2012.

Starting point: a simple model of storage

Dynamics:

$$\underbrace{S^{t+1}}_{\text{SOC at } t+1} = \underbrace{S^{t}}_{\text{SOC at } t} + \underbrace{U^{t}}_{\text{Power at } t}$$

Energy capacity:

$$0 \leq S^t \leq C^t$$

Note:

- Time varying capacities for load aggregations.
- Power capacity, leakage, and injection/extraction losses omitted for simplicity.
- Nonlinear features (transmission losses, reactive power from storage inverters) accommodable with *convex power flow relaxations* (Taylor 2015a).

Multiperiod optimal power flow with storage

Minimize generation cost:	$\sum_{i,t} F_i^t\left(P_i^t\right)$ *subject to*	
Nodal power balance:	$P_i^t = U_i^t + \sum_j B_{ij}\left(\theta_i^t - \theta_j^t\right)$	(1)
Transmission capacity:	$B_{ij}\left(\theta_i^t - \theta_j^t\right) \leq L_{ij}$	(2)
Energy capacity:	$0 \leq S_i^t \leq C_i^t$	(3)
Storage dynamics:	$S_i^{t+1} = S_i^t + U_i^t$	

Lagrange duality, again:

- λ_i^t is the multiplier of (1).
- μ_{ij}^t is the multiplier of (2).
- χ_i^t is the upper multiplier of (3).

Consequences of *storage congestion* are similar to transmission:

- System operator makes money (undesirable).
- High prices (spikes) for loads.
- Generators lose sales.

Arithmetic with KKT of MOPF yields:

$$\underbrace{\sum_t \sum_i \lambda_i^t P_i^t}_{\text{SO budget}} + \underbrace{\sum_{ij} \mu_{ij}^t L_{ij}}_{\text{Transmission right payments}} + \underbrace{\sum_i \chi_i^t C_i^t}_{???} = 0$$

What does the latter term tell us this time?

Storage rights

The SO has a budget surplus of $\sum_t \sum_{ij} \mu_{ij}^t L_{ij} + \sum_i \chi_i^t C_i^t$.

The first term corresponds to Flowgate Transmission Rights.

Definition: The holder of an Energy Capacity Right is entitled to collect $\chi_i^t C_i^{t,k}$, $0 \leq C_i^{t,k} \leq C_i^t$ from SO in each time period.

- Purchasable by loads and generators through right auctions.
- If $\sum_k C_i^{t,k} = C_i^t$ for all i and t, balances SO budget.
- Generators capture more profit, loads *hedge* against price spikes.

Other storage rights:

- *Power Capacity Right* from multiplier of the power constraint
- *Financial Storage Right* = Energy Capacity Right + Power Capacity Right

Storage rights

The SO has a budget surplus of $\sum_t \left(\sum_{ij} \mu_{ij}^t L_{ij} + \sum_i \chi_i^t C_i^t \right)$.

The first term corresponds to Flowgate Transmission Rights.

Definition: The holder of an Energy Capacity Right is entitled to collect $\chi_i^t C_i^{t,k}$, $0 \leq C_i^{t,k} \leq C_i^t$ from SO in each time period.

- Purchasable by loads and generators through right auctions.
- If $\sum_k C_i^{t,k} = C_i^t$ for all i and t, balances SO budget.
- Generators capture more profit, loads *hedge* against price spikes.

Other storage rights:

- *Power Capacity Right* from multiplier of the power constraint
- *Financial Storage Right* = Energy Capacity Right + Power Capacity Right

Storage rights

The SO has a budget surplus of $\quad_t \quad_{ij} \mu_{ij}^t L_{ij} + \quad_i \chi_i^t C_i^t$.

The first term corresponds to Flowgate Transmission Rights.

Definition: The holder of an Energy Capacity Right is entitled to collect $\chi_i^t C_i^{t,k}$, $0 \leq C_i^{t,k} \leq C_i^t$ from SO in each time period.

- Purchasable by loads and generators through right auctions.
- If $\quad_k C_i^{t,k} = C_i^t$ for all i and t, balances SO budget.
- Generators capture more profit, loads *hedge* against price spikes.

Other storage rights:

- *Power Capacity Right* from multiplier of the power constraint
- *Financial Storage Right* = Energy Capacity Right + Power Capacity Right

Further thoughts

Financial storage rights are essential for passive storage. Does passive storage make sense?

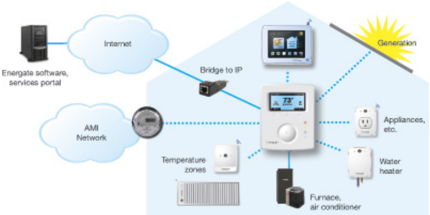
- Invest in storage without dealing with electricity markets.
- Lends itself better to direct operation by SO.
- Insurance against price volatility for loads and generators.
- Add flexibility to regulatory environment.

Part two: demand response

- A handful of ways to fit storage in markets.
- Storage and DR provide similar services.
- How can we represent DR like (but not identically as) storage?

Joint work with Suhail Barot

Why demand response?

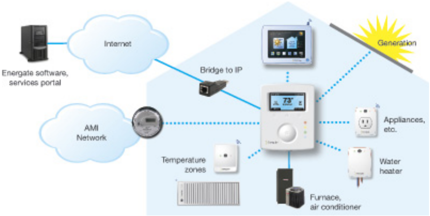


Energate home energy management

Advantages of load-based resources

- Cost: Comm. panel, smart-appliance... 'information' not 'power' sensor & actuator hardware
- Response precision/speed: Buildings: 3-8 min. (J. Mathieu, Gadgil, et al. 2010), Air Cond.: $\sim$ 1 min (Eto et al. 2009), instantaneous in theory ... A generator can take hours!

Why demand response?



Energate home energy management

DR is already here

- 72GW in US (FERC, 2012)
- FERC order #745: pay DR like generators
- Now hundreds of companies

Advantages of load-based resources

- Cost: Comm. panel, smart-appliance... ‘information’ not ‘power’ sensor & actuator hardware
- Response precision/speed: Buildings: 3-8 min. (J. Mathieu, Gadgil, et al. 2010), Air Cond.: $\sim$ 1 min (Eto et al. 2009), instantaneous in theory ... A generator can take hours!

Representing DR

DR and storage can do similar things

- Load-shifting, curtailment, regulation
- Should include DR in operations & planning, like storage

Naive approach

Model all loads.

- Millions of new variables & constraints
- Not ISO's jurisdiction

Our objective

DR representation suitable for:

- Optimal power flow
- T & G planning
- Any optimization w/ power flow

Representing DR

DR and storage can do similar things

- Load-shifting, curtailment, regulation
- Should include DR in operations & planning, like storage

Naive approach

Model all loads.

- Millions of new variables & constraints
- Not ISO's jurisdiction

Our objective

DR representation suitable for:

- Optimal power flow
- T & G planning
- Any optimization w/ power flow

Representing DR

DR and storage can do similar things

- Load-shifting, curtailment, regulation
- Should include DR in operations & planning, like storage

Naive approach

Model all loads.

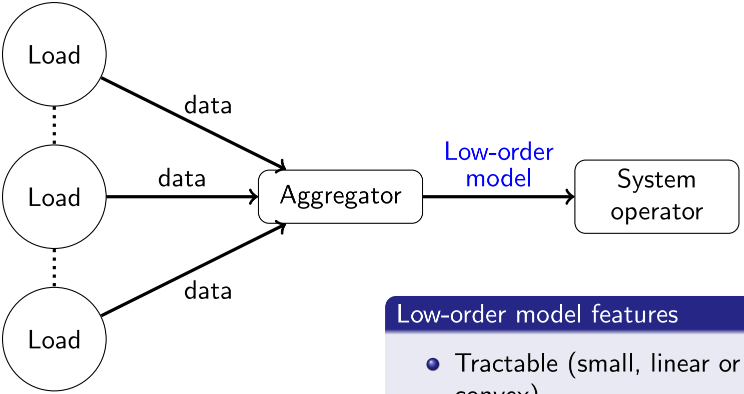
- Millions of new variables & constraints
- Not ISO's jurisdiction

Our objective

DR representation suitable for:

- Optimal power flow
- T & G planning
- Any optimization w/ power flow

Concisely representing DR



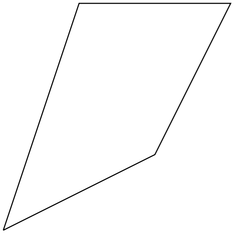
Low-order model features

- Tractable (small, linear or convex)
- Generic (diversity of loads)
- Simple to implement

Generic approach: polytopes

Individual load model

- $x(t)$: power use at time $t = 1, \ldots, T$
- $Ax \leq b$: T-dimensional polytope
- Bounded ... finite power consumption



Examples

Power & ramp limits

- $P_{\min} \leq x(t) \leq P_{\max}$
- $R_{\min} \leq x(t) - x(t-1) \leq R_{\max}$

Deferrable loads w/ arrivals and departures

- ${}_{t=1}^{T} x(t) = E$, $x(t) \geq 0$
- $x(t) = 0$ for $t \notin \{a, d\}$

Also: Input/output/leakage losses, storage, TCLs, most non-quantized loads ...

Suppose I have many loads:

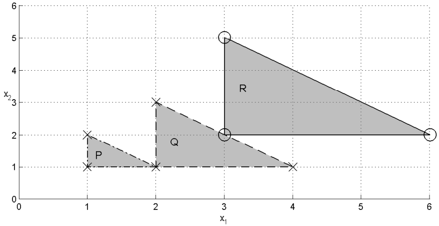
$$X_i = \{x_i \mid A_i x_i \le b_i\}, \quad i = 1, \dots, N.$$

If $N = 10^6$, TN new variables, lots of constraints ... too much info.

Aggregate capabilities:

$$P = \left\{ p \;\middle|\; p = \sum_{i=1}^{N} x_i,\; x_i \in X_i,\; i = 1, \dots, N \right\}.$$

P is called the Minkowski sum of X_i, $i = 1, \dots, N$.



- Left: the MS of two triangles.
- $\mathrm{Dim}(P) = T$.
- Goal: A concise approximation of P.

The Minkowski sum

Suppose I have many loads:

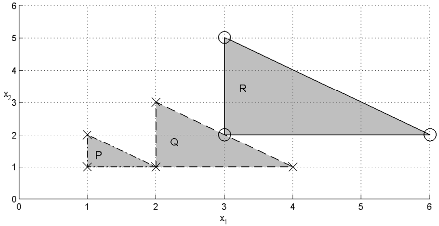
$$X_i = \{x_i \mid A_i x_i \leq b_i\}, \quad i = 1, \dots, N.$$

If $N = 10^6$, TN new variables, lots of constraints ... too much info.

Aggregate capabilities:

$$P = \left\{ p \;\middle|\; p = \sum_{i=1}^{N} x_i,\ x_i \in X_i,\ i = 1, \dots, N \right\}.$$

P is called the Minkowski sum of X_i, $i = 1, \dots, N$.



- Left: the MS of two triangles.
- $\mathrm{Dim}(P) = T$.
- Goal: A concise approximation of P.

Prior work

A few existing approaches:

- Using PDEs, probability: Malhamé and Chong 1985, D. S. Callaway 2009, Perfumo et al. 2012.
- As effective storage (a polytope): TCLs in J. L. Mathieu et al. 2013; J. Mathieu, Kamgarpour, et al. 2015, Hao et al. 2013; EVs in Nayyar et al. 2013.
- Observed Minkowski sum structure: Hao et al. 2013, Alizadeh et al. 2014 (plasticity).

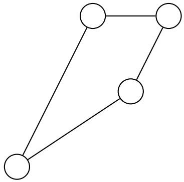
Our contribution

- Use Minkowski sum mechanistically
- Admits all convex, closed polytopes
- Relies only on LP, matrix algebra

Challenge: computational complexity

Polytopes (convex & closed) can be specified in two ways:

- Facets: $Ax \leq b$ (each row is a facet)
- Vertices: convex hull of points in $\mathbb{R}^T$



Loads almost always modeled as facets.

Typical Minkowski sum procedure:

1. Convert facets to vertices (vertex enumeration, NP-hard, Khachiyan et al. 2008)
2. Minkowski sum in vertex representation (polynomial-time in # of vertices)
3. Convert vertices to facets (also hard)

No known efficient algorithm for Minkowski sum in facet representation.

Special case: same A matrices.

$$X_1 = \{x \mid Ax \leq b_1\}, \quad X_2 = \{x \mid Ax \leq b_2\}.$$

Define:

$$Q = \{x \mid Ax \leq b_1 + b_2\}$$

Proposition

Q contains the Minkowski sum of X_1 and X_2.

Proof: Suppose z is in the Minkowski sum of X_1 and X_2. Then there exist $x_1 \in X_1$ and $x_2 \in X_2$ such that $z = x_1 + x_2$. By construction, $A(x_1 + x_2) \leq b_1 + b_2$ and therefore $z \in Q$, i.e., any element of the Minkowski sum is in Q.

Can we generalize this?

Outer approximation of the Minkowski sum

Special case: same A matrices.

$$X_1 = \{x \mid Ax \leq b_1\}, \quad X_2 = \{x \mid Ax \leq b_2\}.$$

Define:

$$Q = \{x \mid Ax \leq b_1 + b_2\}$$

Proposition

Q contains the Minkowski sum of X_1 and X_2.

Proof: Suppose z is in the Minkowski sum of X_1 and X_2. Then there exist $x_1 \in X_1$ and $x_2 \in X_2$ such that $z = x_1 + x_2$. By construction, $A(x_1 + x_2) \leq b_1 + b_2$ and therefore $z \in Q$, i.e., any element of the Minkowski sum is in Q.

Can we generalize this?

Outer approximation: general case

Now suppose

$$X_1 = \left\{ x \;\; \begin{bmatrix} A_c \\ A_1 \end{bmatrix} x \le \begin{bmatrix} b_{c1} \\ b_1 \end{bmatrix} \right\}, \quad X_2 = \left\{ x \;\; \begin{bmatrix} A_c \\ A_2 \end{bmatrix} x \le \begin{bmatrix} b_{c2} \\ b_2 \end{bmatrix} \right\}.$$

A_c are common rows in the A matrices. Define

$$A = \begin{bmatrix} A_c \\ A_1 \\ A_2 \end{bmatrix}$$

We can choose $\hat{b}_1$ and $\hat{b}_2$ so that

$$X_1 = \left\{ x \;\; Ax \le \begin{bmatrix} b_{c1} \\ b_1 \\ \hat{b}_1 \end{bmatrix} \right\}, \quad X_2 = \left\{ x \;\; Ax \le \begin{bmatrix} b_{c2} \\ \hat{b}_2 \\ b_2 \end{bmatrix} \right\}.$$

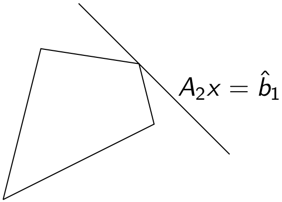
General outer approximation:

$$P = \left\{ z \;\; Az \le \begin{bmatrix} b_{c1} + b_{c2} \\ b_1 + \hat{b}_1 \\ \hat{b}_2 + b_2 \end{bmatrix} \right\}.$$

Algorithm

LP for the smallest $\hat{b}$:

$$\hat{b}_1 = \max_x \; A_2 x \quad \text{s.t.} \quad \begin{matrix} A_c \\ A_1 \end{matrix} \; x \le \; \begin{matrix} b_{c1} \\ b_1 \end{matrix}$$



Procedure:

1. Assemble all unique rows of the A_i, $i = 1, \ldots, N$, construct common A matrix.
2. Use LP to find $\hat{b}_i$'s, construct new b_i vectors, $i = 1, \ldots, N$.
3. Outer approximation is $\left\{ z \quad Az \le \quad {}_{i=1}^{N} b_i \right\}$.

Theoretical results

The outer approximation is **exact** for

- loads with only power constraints (hypercubes):

$$p_{i,\min}(t) \leq x_i(t) \leq p_{i,\max}(t) \quad i = 1, \dots, N, \;\; t = 1, \dots, T;$$

- deferrable loads (simplices):

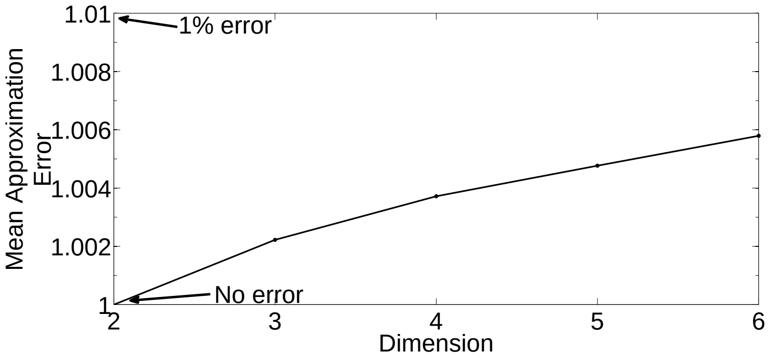
$$\sum_{t=1}^{T} x_i(t) = E_i \quad i = 1, \dots, N.$$

Comparable to existing results for deferrable loads w/ arrivals & departures.[3]

[3] A. Nayyar et al. "Aggregate flexibility of collections of loads". In: *Decision and Control (CDC), IEEE 52nd Annual Conference on*. Invited. Dec. 2013, pp. 5600–5607. DOI: 10.1109/CDC.2013.6760772.

Empirical performance: loads with power & energy constraints

$$\text{Error} = \frac{\textsf{Volume of Approximation}}{\textsf{Volume of Exact Minkowski Sum}}$$



Mean error over 1,000 random pairs for each number of dimensions (time horizon)

Outlook

Summary

- Many loads are modeled by polytopes.
- Aggregate flexibility is the Minkowski sum ... computationally intractable.
- Our work: a generic, tractable, accurate outer approximation.

Future work

- Outer approximation for loads defined SOCP & SDP chance constraints to accommodate uncertainty, e.g., arrival & departures, unknown model parameters.
- Can we define financial storage rights for general polytope or convex resources? Almost certainly ...

Questions?

J.A. Taylor. “Financial storage rights”. In: *Power Systems, IEEE Transactions on* 30.2 (Mar. 2015), pp. 997–1005. DOI: 10.1109/TPWRS.2014.2339016

S.F. Barot and J.A. Taylor. “A concise, approximate representation of a collection of loads described by polytopes”. In: *Under review* (2015)

josh.taylor@utoronto.ca